Abridgment

Reliability of Slopes: Incorporating Qualitative Information

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The transition from theoretical results to real results is often the critical step in the decisionmaking process of a geotechical engineer. The proposed method for the reliability analysis of slopes calculates the theoretical solution and then modifies it to account for qualitative information. The first step involves calculation of the probability of failure on the basis of available information from the idealized geotechnical structure. This theoretical probability is then modified by a quality factor to yield an actual probability of failure. Qualitative aspects are represented by verbal statements that are translated to belief/importance factors in the form of membership functions; the processing of this information is based on fuzzy logic. The results of corrected probabilities of failure are compared with experience-based predictions made by Lambe in his Terzaghi Oration at the Eleventh Conference of the International Society for Soil Mechanics and Foundation Engineering. Data from sociological studies and questionnaire-based measurements of risk acceptance are presented. The corrected probability of failure is then compared with the membership function of the acceptable risk to establish a measure of the urgency of repairs. The approach is implemented in a computerized decision support system incorporating extensive support information and recommendations.

In his Terzaghi Oration given at the Eleventh International Conference, Lambe presented a figure that relates probability of failure to factor of safety (*1*). It is not based on calculations, but a numerical representation of engineering judgment. It incorporates qualitative factors that relate to design, analysis, construction, and performance of a geotechnical system, such as the Amuay Project.

A method to incorporate qualitative information in the standard reliability analysis of slopes is presented. The approach separates the theoretical solution based on probability theory from the practical one, obtained from the theoretical solution by incorporating qualitative information by means of fuzzy set theory.

THEORETICAL SOLUTION

Nominal Factor of Safety

In common practice, the value of resistance selected for the analysis is lower than the mean resistance, whereas the reverse is true for the load. The selected values are not absolute lower and upper bounds, respectively, however; they are conservative estimates based on experience.

The conservative estimation of design parameters results in the calculation of a nominal factor of safety, FS_{nom}, which is smaller than the mean factor of safety FS_μ. Assuming normal distribution for the load L and resistance R, the relation between FS_μ and FS_{nom} is

$$FS_{\text{nom}} = FS_\mu \left(\frac{1 - \delta_R V_R}{1 + \delta_L V_L}\right) \qquad (1)$$

where

V = coefficient of variation, and
δ = the standardized value that allows for a certain percentile under the curve (e.g., for the 90th-percentile load and resistance, $\delta_R = \delta_L = 1.3$).

The coefficient of variation for the resistance is about 0.2 when effective stress analysis is applied to a slope in homogeneous media and it is higher for soils with some cementation. The coefficient of variation for the resistance varies between 0.15 and 0.25 for normal load conditions. On the basis of this equation, the ratio between the two factors of safety is about 1.5 to 2.0 in common slope stability practice.

Theoretical Probability of Failure

If the safety margin, SM, is defined as the difference between resistance R and load L, the probability of failure p_f is the probability of $SM \leqq 0$. Assuming normal distributions for both R and L, p_f is

$$p_f = \Phi \left(\frac{FS_\mu - 1}{V_R^2 FS_\mu^2 + V_L^2}\right) \qquad (2)$$

where Φ = the standard normal distribution. Other distributions may be assumed without affecting the rest of the analysis.

ADJUSTED PROBABILITY OF FAILURE

The consideration of variables not included in the theoretical analysis is usually viewed with apprehension because of their vague and qualitative nature. To circumvent this problem,

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qualitative information is included in the analysis by means of fuzzy sets (2). Although this is not the only possible method to incorporate qualitative information, it was found convenient for the problem considered.

Qualitative aspects are represented by verbal statements that are transformed to belief/importance factors on a selected scale. The result of this analysis is a fuzzy adjusted probability. If the result of the theoretical probability of failure is expressed as

$$p_f = 10^{-\beta} \quad (3)$$

then the adjusted value may be given in the form (3)

$$p_f^c = 10^{-\beta\alpha} \quad (4)$$

where α is a correction factor of qualitative parameters not considered in the theoretical analysis. When qualitative aspects indicate very poor conditions of the project, α may be a small number. In the extreme case, $\alpha = 0$, and the system has an adjusted probability of failure of 1.0. In the opposite case, when all aspects of the project are extremely good, α approaches 1.0 and the adjusted probability of failure p_f^c remains equal to the theoretical value p_f.

Fuzzy Set Representation

The α correction factor is evaluated using fuzzy sets (4). A unique representation called supportless fuzzy sets is used in this analysis. It consists of a list of membership values defined at discrete points.

Stacks of Fuzzy Constraints

Six categories were selected to represent the possible levels for quality of the project: excellent operation, sound operation, intermediate operation, approximate operation, norational operation, and very good service conditions. The last category allows for the compensatory effect of very good performance and maintenance on the lesser quality of other parameters.

Each category is defined as a group of constraints that restrict the conditions of a project must have to belong to such a category. These dimensions and their constraints result in a data structure referred to as the stack of contraints. The stack of constraints for very good service conditions follows:

Qualifications of the engineer-designer
(0.4 0.8 1.0 0.9 0.4 0.2 0.1)
Extent/quality of geologic assessment
(0.4 0.8 1.0 0.9 0.4 0.2 0.1)
Quality of available data
(0.2 0.8 1.0 0.9 0.2 0.1 0.0)
Quality of design method
(0.4 0.8 1.0 0.9 0.4 0.2 0.1)
Completeness of the design of the structure
(0.0 0.1 0.2 0.7 1.0 1.0 1.0)
Importance of design errors or emissions
(0.2 0.8 1.0 0.9 0.2 0.1 0.0)
Contractor's prior record
(0.4 0.8 1.0 0.9 .04 0.2 0.1)
Supervision during construction
(0.2 0.8 1.0 0.9 0.2 0.1 0.0)
Quality of field controls during construction
(0.4 0.8 1.0 0.9 0.4 0.2 0.1)
Importance of construction errors
(0.2 0.8 1.0 0.9 0.2 0.1 0.0)
Difficulties during construction
(0.4 0.8 1.0 0.9 0.4 0.2 0.1)
Monitoring program
(0.0 0.0 0.0 0.2 0.5 0.8 1.0)
In-service inspection
(0.0 0.0 0.0 0.2 0.5 0.8 1.0)
Manfunctions during the life of the structure
(0.0 0.0 0.0 0.2 0.5 0.8 1.0)
Maintenance program
(0.0 0.0 0.0 0.2 0.5 0.8 1.0)

To determine the similarity between a given Category A and the project, the characteristics of the project are compared with those constraints that represent Category A. Comparison at the level of each dimension i is based on the filtering operation, which results in a similarity value s_i (5),

$$S_i^A = \frac{\text{Cardinality } (mf_i^{proj} \cap mf_i^A)}{\text{Cardinality } (mf_i^{proj})} \quad (5)$$

where mf = membership function. The overall similarity between the project and Category A is the minimum of all individual similarities S_i^A.

ACCEPTABLE PROBABILITY OF FAILURE

The following parameters were found to determine the level of acceptable risk in the context of slope stability (6–8):

- Loss of human life,
- Potential economic loss,
- Relative cost of lowering the probability of failure (certain) with respect to the expected cost of postfailure repairs (probable),
- Technical and economic capacity to implement repairs,
- Unique structure, or one of a group,
- Existing or to-be-constructed structure,
- Temporary or permanent,
- Remaining service life,
- Type and importance of service, and,
- Effect on lifelines.

Different sources of information were used to determine the levels of acceptable p_f for slopes. Table 1 summarizes the results of a questionnaire answered by the engineers involved in the reliability of existing slopes. These data support more general results obtained by the authors from two different groups of assessors, including professors and students from a variety of engineering branches. In Study 1, 22 assessors evaluated the acceptability of a generic failure (4), whereas in Study 2, 8 assessors evaluated the acceptability of the failure of temporal and permanent structures of either low or high

TABLE 1 SLOPE STABILITY—ACCEPTABLE PROBABILITY OF FAILURE

Conditions	P_f
Unacceptable in most cases	less than 10^{-1}
Temporary structures no potential life loss low repair cost	10^{-1}
Nil consequences of failure high cost to lower Pf i.e., bench slope, open pit mine	1 to 2 10^{-1}
Existing slope of riverbank at docks available alternative docks repairs can be promptly done do-nothing: attractive idea	5 10^{-2}
To-be-constructed; same condition	less than 5 10^{-2}
Slope of riverbank at docks no alternative docks pier shutdown threatens operations	1 to 2 10^{-2}
Low consequences of failure repairs can be done when time permits repair cost < cost to lower Pf	10^{-2}
Existing large cut interstate highway	1 to 2 10^{-2}
To-be-constructed; same condition	less than 10^{-2}
Acceptable in most cases except if lives may be lost	10^{-3}
Acceptable for all slopes	10^{-4}
Unnecessarily low	10^{-5} or lower

importance. The results of Study 2 show that temporal and low importance systems are represented by the same curve, as are the results for permanent and high importance systems. Membership functions from these two studies are shown in Figure 1. These results are in agreement with observations by Ashby in the environmental field (*9*).

THE PROGRAM SLOPE

SLOPE consists of seven blocks.

Block 1: Input of Preliminary Information

Block 2: Verification of the Calculated Factor of Safety

The purpose of this part of the program is to aid the user in assessing the validity of the stability analysis. In response, the system may recommend that the user review the analysis before proceeding. In this part, questions will address the following design decisions.

- Selection of strength parameters: short and long term, effective stress or total stress analysis, brittleness, heterogeneity and strain compatibility, in situ and induced pore pressure, variability of soil parameters, and tests' scale effects.
- Possibility of weak seams (historical evidence or resulting from in situ testing).
- Formation of tension cracks and buildup of pore pressure.
- Selection of failure surface.
- Potential effects of erosion.
- Other loads.

Block 3: Selection of Acceptable Risk

This section gathers external information about the project in order to determine the acceptable risk. The user's quali-

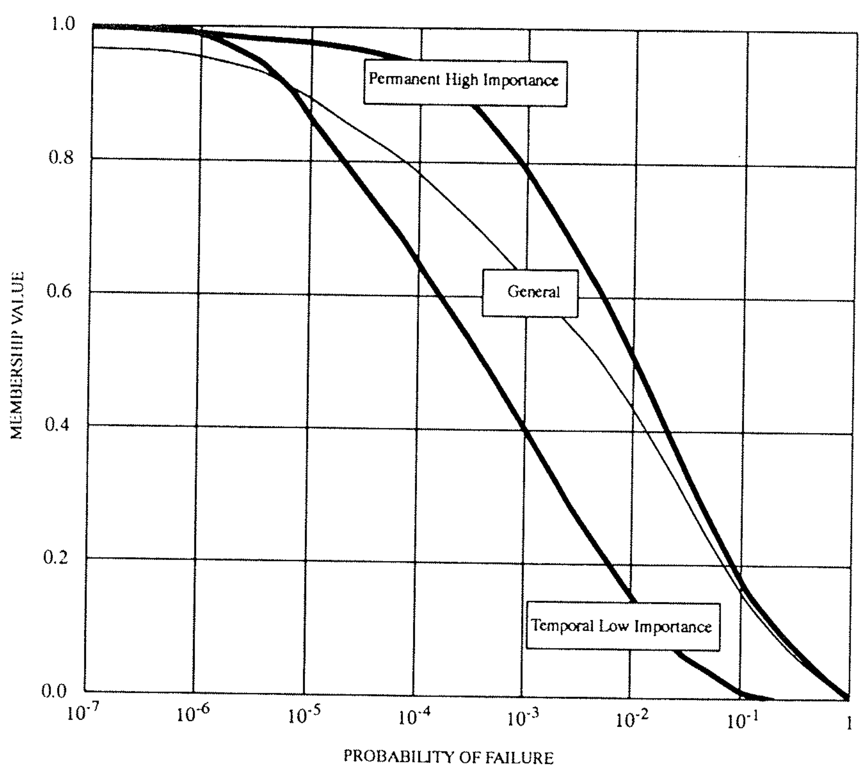

FIGURE 1 Membership functions for acceptable probability of failure.

tative answer is translated into a fuzzy set and compared with the corresponding constraint in three stacks. Each stack represents a different level of acceptable risk: very low, intermediate, and very high. There is a membership function for acceptable risk associated with each stack. Once the similarity between the project and each stack is established, S_i, the membership function for the acceptable risk, is calculated. The computational formula is shown in Equation 6.

$$\nu = \nu_{vl} * S_{vl}^2 + \nu_{in} * S_{in}^2 + \nu_{vh} * S_{vh}^2 \quad (6)$$

where

ν_{vl} = very low risk,
ν_{in} = intermediate risk, and
ν_{vh} = very high risk.

This approach allows for compensatory effects and gives emphasis to the best-matched category.

Block 4: Calculation of Theoretical Probability of Failure

The calculation of the mean factor of safety FS_μ follows Equation 1. The theoretical probability of failure is then calculated by means of a polynomial approximation.

Block 5: Selection of Membership Function for α Correction

The user's qualitative answers are translated into fuzzy sets and compared with the corresponding constraint in each of the stacks. Each stack or category has an associated membership function for the value of α (α ranges between 0.0 and 1.0, and is discretized every 0.1). The final similarity value of each category is used to calculate the membership function for the project, following the same approach used to determine the membership function for acceptable risk (Equation 6).

Blocks 6 and 7: Adjusted Probability of Failure and Urgency of Repairs

The adjusted p_f is obtained by the fuzzy multiplication of β and α, resulting in a non-crisp value p_f^c. Finally, p_f^c is filtered through the membership function for the acceptable risk to obtain a final acceptability index *AI* (a crisp number):

$$AI = \frac{\text{Cardinality } (mf_{\alpha\beta} \cap mf_{\text{risk}})}{\text{Cardinality } (mf\alpha\beta)} \quad (7)$$

The complement of *Ai* is a measure of the need for immediate repairs, in other words, an urgency index *UI*.

RESULTS AND OBSERVATIONS

Results obtained with this type of analysis were compared with corrections proposed by Lambe (*1*), observing very similar trends. Additional results that further support the importance of qualitative variables on performance and emphasize the need to design for low theoretical probability of failure are shown in Figure 2. For a particular design with a central factor of safety of 2.0 and a theoretical p_f less than 10^{-5}, a project of intermediate quality will bring the adjusted probability of failure to a value greater than 10^{-3} (note that low coefficients of variation are assumed). Social limits on the probability of failure fuzzified from Ashby (*9*) and membership values for acceptable probability of failure were added to Figure 2. It can be concluded that in order to obtain an

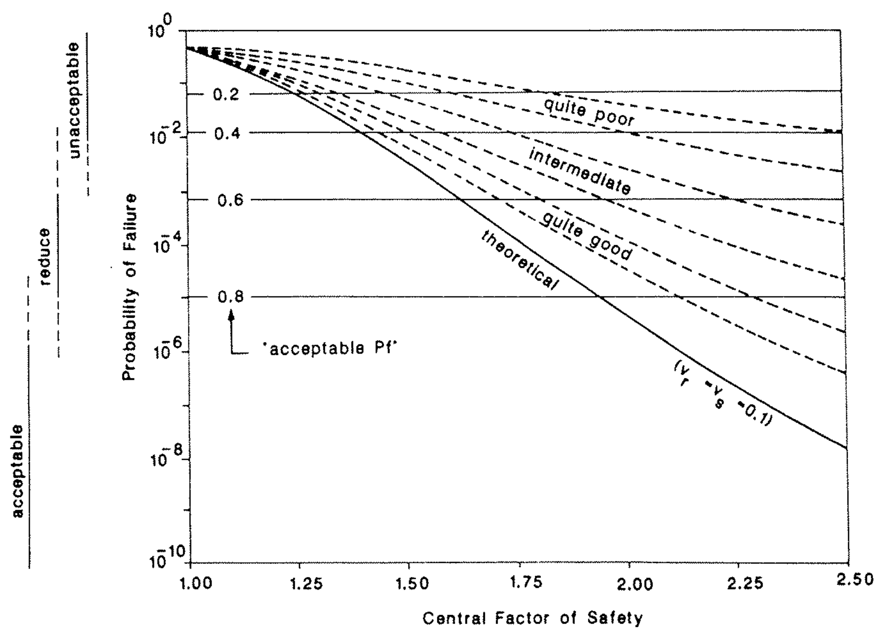

FIGURE 2 Results and implications.

adjusted probability of failure of about 10^{-4} with standard practice, a minimum common practice *FS* of about 1.6 to 1.7 is needed.

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